Building Period Formulas for Estimating Seismic Displacements

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Traditionally, empirical formulas for building period recommended for code applications are intentionally calibrated to underestimate the period in order to estimate base shear conservatively. At a shorter period, the seismic displacements are smaller, however, and hence underestimated. This note discusses this issue and recommends formulas for estimating seismic displacements of buildings.

INTRODUCTION

The fundamental vibration period of a building appears in the equation specified in building codes to calculate the design base shear and lateral forces. To estimate the period, building codes provide empirical formulas that depend on the building material (steel, R/C, etc.), building type (frame, shear wall, etc.), and overall dimensions. These formulas are typically calibrated to underestimate the periods so that the pseudo-acceleration (Figure 1) and seismic forces are overestimated. At a shorter period, the seismic displacements are smaller (Figure 1), however, and hence underestimated. The purpose of this note is to discuss this issue and recommend formulas for estimating seismic displacements of buildings.

![Figure 1. Elastic pseudo-acceleration and deformation spectra for ground motions with $\ddot{u}_g = 1g$, $\dot{u}_g = 122\text{cm/sec}$ and $u_g = 91.4\text{cm}$.

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MOMENT-RESISTING FRAME BUILDINGS

Obviously, any empirical formula should be based on periods of buildings “measured” from their motions recorded during earthquakes. The most relevant data are from structures shaken strongly but not deformed significantly into the inelastic range. Figures 2 and 3 show such data for 42 Steel Moment-Resisting Frame (MRF) buildings and 27 Reinforced-Concrete (R/C) MRF buildings in California derived from their recorded motions during several earthquakes starting with the 1971 San Fernando earthquake (Goel and Chopra, 1997b); the measured periods in two orthogonal lateral directions are shown by circles connected by a vertical line. The best-fit curve, labeled as $T_R$, comes from regression analysis to determine $\alpha$ and $\beta$ in the equation relating period $T$ to height $H$ of the building: $T = \alpha H^\beta$. Also included are two other curves: $T_L$ which represents the best-fit minus one standard deviation curve (referred to as the best-fit $-1\sigma$ curve), and $T_U$, the best-fit plus one standard deviation curve (referred to as the best-fit $+1\sigma$ curve). These figures give an impression of the scatter in the data of the measured periods relative to the curves from regression analysis. As expected, the data fall above and below the best-fit curve, more or less evenly, and most of the data are above the best-fit $-1\sigma$ curve and below the best-fit $+1\sigma$ curve.

![Figure 2](image-url)

**Figure 2.** Results of regression analysis for Steel MRF buildings.

Traditionally, empirical formulas for building period recommended for code applications are intentionally calibrated to underestimate the period in order to estimate base shear conservatively. Based on the “measured” data, the best-fit $-1\sigma$ curves were recommended for estimating the building period (Goel and Chopra, 1997a):

\[
T_L = 0.028H^{0.80} \quad \text{Steel Buildings} \\
T_L = 0.016H^{0.90} \quad \text{R/C Buildings}
\]

where $H$ is the height of the building in feet. These formulas have been compared with those in the Uniform Building Code (Goel and Chopra, 1997a).
However, Equations (1) and (2) may not be appropriate for estimating seismic displacements of buildings. To obtain a conservative estimate of building displacements, the period should be overestimated and the best-fit $+ 1\sigma$ curve is recommended:

$$T_U = 0.045 H^{0.80} \quad \text{Steel Buildings} \quad (3)$$

$$T_U = 0.023 H^{0.90} \quad \text{R/C Buildings} \quad (4)$$

![Figure 3](image)

**Figure 3.** Results of regression analysis for R/C MRF buildings.

**SHEAR WALL BUILDINGS**

Figure 4 shows “measured” periods for 16 R/C Shear Wall (SW) buildings. Based on a theoretical formula, derived using Dunkerley’s method, and regression analysis of these data, the best-fit, best-fit $+ 1\sigma$, and best-fit $- 1\sigma$ curves were developed (Figure 4). The best-fit $- 1\sigma$ curve was recommended (Goel and Chopra, 1998) to estimate base shear conservatively:

$$T_L = 0.0019 \frac{H}{\sqrt{A_e}} \quad \text{SW Buildings} \quad (5)$$

where $A_e$ is the equivalent shear area, expressed as a percentage of the building plan area, $A_B$, and given by:

$$A_e = \frac{100}{A_B} \sum_{i=1}^{N_w} \left( \frac{H_i}{H_i} \right)^2 \frac{A_i}{\left[ 1 + 0.83 \left( \frac{H_i}{D_i} \right)^2 \right]} \quad (6)$$

In Equation (6), $A_i$, $H_i$ and $D_i$ are the area, height, and dimension in the direction under consideration of the $i$th shear wall and $N_w$ is the number of shear walls.

However, to obtain a conservative estimate for building displacements, the best-fit $+ 1\sigma$ curve is appropriate and the following formula is recommended:
\[ T_u = 0.0026 \frac{H}{\sqrt{A_e}} \]  
SW Buildings

**Figure 4.** Results of regression analysis for R/C SW buildings.

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**REFERENCES CITED**

